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Decision Making Procedure Based on Jaccard Similarity Measure with Z-numbers

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ABSTRACT

Fuzzy set with similarity measure approaches are known to be effective in handling imprecise and subjective information to solve decision making problems. Many methods have been introduced based on these two concepts. However, most methods do not take into account the reliability factor of the imprecise information in the evaluation process. In 2010, Zadeh coined the idea of Z-number that has the ability to consider the reliability factor or the level of confidence of human's information expression. Since then, some decision-making methods have included this concept. In this paper, we present a new fuzzy decision making procedure by integrating the Jaccard similarity measure with Z-number to solve a multi criteria decision making problem. The conversion method of the Z-number based linguistic value to trapezoidal fuzzy numbers is used and the Jaccard similarity measure of the expected intervals of trapezoidal fuzzy numbers is applied to obtain the final decision. The feasibility of the methodology is demonstrated by investigating the preference factors that could influence customers to buy their preferred choice of car. The proposed methodology is applicable to solving decision making with a fuzzy environment to achieve a reliable and optimal decision.

Keywords: Decision making, jaccard similarity measure, Z-number, multicriteria group decision making, expected interval of fuzzy numbers

INTRODUCTION

Decision making can be defined as a process of problem solving which results in an action (Easton, 1976). The evaluation process becomes difficult due to uncertainty or incomplete information but decision makers are capable of successfully developing and representing the subjective information. Methods are likely used to arrive at their

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judgment linguistically rather than provide a finite crisp value. Emergence of fuzzy set theory (Zadeh, 1965) and methods for solving decision making problems based on the fuzzy number have been acknowledged. Fuzzy Analytic Hierarchy Process (FAHP) (Kaboli et al., 2007; Bozbura et al., 2007) and Fuzzy TOPSIS (Chen, 2000; Goli, 2013), known to be effective tools for solving complicated multi-criteria decision making problems, have been applied successfully to numerous problems. Another tool for solving similar problems which is also widely applied in fuzzy multicriteria decision making (FMCDM) problem is the similarity measure approach.

Similarity Measure

Essentially, similarity measure (SM) calculates the degree of similarity between objects or sets through comparisons. In cases of imprecisions , similarity measure of fuzzy numbers is used to measure the degree of similarity between the evaluations. This approach has been successfully applied in solving, for instance, emergency management problem (Xu et al., 2012) and quality of river water (Wang et al., 2013).

Different researchers have developed different similarity measure techniques for diverse purposes such as decision making, clustering, risk analysis, image processing and pattern recognition or information retrieval. Hsieh et al. (1999) proposed similarity measure for generalised fuzzy number based on graded mean integration representation distance. Chen and Chen (2003) presented similarity measure based on centre of gravity points of the generalised fuzzy numbers. Yong et al. (2004) introduced a new similarity measure based on radius of gyration to address the shortcomings of Chen and Chen (2003). Sridevi and Nadarajan (2009) proposed a similarity measure for generalised fuzzy numbers which later became a basis for another similarity measure introduced by Farhadinia and Ban (2013) which is suitable for situations where there are two different generalised fuzzy numbers with the same centre of gravity points.

In recent years, the vector based similarity measure has been adopted by several researchers to solve fuzzy decision making problems. Ye (2012a) applied the Dice similarity measure in the form of a vector which incorporates inner products to find similarity between query and document in order to retrieve the document that represent the query. In the same year, Ye (2012b,c) proposed the vector similarity measure between trapezoidal intuitionistic fuzzy numbers for multi-criteria group decision making, in which the criteria weights and the evaluation values in a decision matrix are expressed by trapezoidal intuitionistic fuzzy numbers, followed by the Dice similarity measure based on the expected interval of trapezoidal fuzzy number with unknown criteria weight. Wu and Mendel (2014) proposed a vector similarity measure for interval type-2 and type-1 fuzzy sets to solve the linguistic approximation problem. Through the weight expected similarity measure between each alternative and the ideal alternative, the ranking order of all the alternatives can be determined and the best one (s) is easily identified.

Z-number

Since uncertainty phenomenon is perceived to exist in human decision making, the reliability of information in decision making can be attributed using the Z-number concept. Fundamentally, a Z-number concept incorporates confidence, certainty, strength of belief or sureness factor in the fuzzy environment evaluation (Zadeh, 2011). By definition, a Z-number, is an ordered pair of fuzzy numbers, (*A*,*R*). A Z-number is associated with a real-valued uncertain variable, *X* with the first component, *A*, is a fuzzy restriction on the values of *X*. *A* is referred to possibilistic restriction of *X*. The second component, *R*, is the certainty that *X* is *A* (reliability factor). *A* and *R* are perception-based values which are described in a natural language (Zadeh, 2011; Kang et al., 2012b). Zadeh (2011) remarked that the Z-number is more general than the concept of confidence intervals in probability theory. It has been applied in solving some decision-making problems with multiple uncertain environments (Kang et al., 2012a; Zeinalova, 2014; Gardashova, 2014).

In this paper, a new decision making procedure will be presented by using the weighted expected Jaccard similarity measure based on Z-numbers which will be used to solve multi-criteria group decision making (MCGDM) problems. A case study investigates the effectiveness of the proposed decision making procedure.

PRELIMINARIES

This section briefly introduces some definitions and basic concepts related to fuzzy sets, fuzzy numbers, Z-number and expected Jaccard similarity measure of the trapezoidal fuzzy number.

Definition 1

(Bojadziev and Bojadziev, 2007) A fuzzy set, *A*, is defined by a set or ordered pairs, binary relation on a universe *X* which may be given as:

$$\{(x, \mu_A(x); x \in X\}$$

where $\mu_A(x)$ is known as membership function in the interval [0,1] that specifies the grade or degree to which any element *x* in *X* belongs to the fuzzy set *A*.

Definition 2

(Allahviranloo et al., 2012) A generalised trapezoidal fuzzy number GTpFN is defined as $A = (a_1, a_2, a_3, a_4, w_A)$ with membership function $\mu_A(x) : R \to [0,\ 1]$ denoted as:

$$\mu_A(x) = \begin{cases} 0, & x < a_1 \\ w_A(x - a_1)/a_2 - a_1, & a_1 \le x \le a_2 \\ w_A, & a_2 \le x \le a_3 \\ w_A(a_3 - x)/a_4 - a_3, & a_3 \le x \le a_4 \\ 0, & x < a_4 \end{cases}$$

where, $a_1 \le a_2 \le a_3 \le a_4$, $a_1, a_2, a_3, a_4 \in R$ and $w_A \in [0,\ 1]$. When $w_A = 1$, the GTpFN becomes a regular trapezoidal fuzzy number. If $w_A = 1$ and $a_2 = a_3$, then it becomes a triangular fuzzy number.

Definition 3

(Kang et al., 2012). The expectation of a fuzzy number is denoted as:

$$E_A(x) = \int_x x\mu_A(x)\ dx$$

which is considered as the information strength supporting the fuzzy set *A*. Note that this value is not the same as the meaning of the expectation of probability space.

Definition 4

(Zadeh, 2011). A Z-number is an ordered pair of two fuzzy numbers denoted as *Z*=(*A*,*R*). The first component of the fuzzy number, *A*, is a restriction on the values of real-valued uncertain variable, *X*. Meanwhile, the second component of the fuzzy number, *R*, is a measure of reliability such as confidence, sureness, strength of belief, probability or possibility for the first component.

Expected Interval of Generalised Trapezoidal Fuzzy Number (GTpFN) (Ye, 2012c)

Let $A = (a_1, a_2, a_3, a_4; w_A)$ be a GTpFN with strictly monotonic left sides and right sides given as $f_A(x) = w_A(x - a_1)(\ a_2 - a_1)$, $g_A(x) = w_A(x - a_4)(\ a_3 - a_4)$ respectively. Their inverse functions are defined as $f_A^{-1}(x) = a_1 + (a_2 - a_1)(y / w_A)$, $g_A^{-1}(x) = a_4 + (a_3 - a_4)(y / w_A)$ respectively where $y \in [0, w_A]$. The expected interval of the GTpFN *A* is a crisp interval $\boldsymbol{E}\ (A)$ given by

$$EI(A) = [E_L(A), E_U(A)] = \left[\int_0^{w_A} f_A^{-1}(x)dy, \int_0^{w_A} g_A^{-1}(x)dy\right] ..$$

It can be shown that

$$E_L(A) = \int_0^{w_A} f_A^{-1}(x)dy = \int_0^{w_A}\left[a_1 + (a_2 - a_1)\frac{y}{w_A}\right]dy \quad = a_1 w_A + \frac{a_2 - a_1}{2} w_A = \frac{w_A(a_1 + a_2)}{2} .$$

Similarly, we have $E_U(A) = \frac{w_A(a_3 + a_4)}{2}$. Hence, the expected interval of GTpFN $A = (a_1, a_2, a_3, a_4; w_A)$ is $\boldsymbol{E}\ (A) = [w_A(a_1 + a_2)/2, w_A(a_3 + a_4)/2]$ and the expected value is defined as the centre of the expected interval of A given by

$$E(A) = (E_L(A) + E_U(A)\ /2 = w_A(a_1 + a_2 + a_3 + a_4)/4 \ .$$

Jaccard Similarity Measure between Vectors

Let $X=(x_1,x_2,...,x_n)$ and $Y=(y_1,y_2,...,y_n)$ be the two vectors of length *n*, where all the coordinates are positives. The Jaccard index (Wu and Mendel, 2014) that measures the similarity of these vectors is defined as:

$$J=\frac{X\cdot Y}{\|X\|_2^2+\|Y\|_2^2}=\frac{\sum_{i=1}^{n}x_iy_i}{\sum_{i=1}^{n}x_i^2+\sum_{i=1}^{n}y_i^2-\sum_{i=1}^{n}x_iy_i}$$

where $X.Y=\sum_{i=1}^{n}x_iy_i$ is the inner product of the two vectors *X* and *Y*, and $\|X\|_2=\sqrt{\sum_{i=1}^{n}x^2}$, $\|Y\|_2=\sqrt{\sum_{i=1}^{n}y^2}$ are the Euclidean norms (L_2) of *X* and *Y*. Let $A_1=(a_{11},a_{12},a_{13},a_{14};w_1)$ and $A_2=(a_{21},a_{22},a_{23},a_{24};w_2)$ be two GTFNs. The expected intervals of two GTpFNs A_1 and A_2 are $EI(A_1)=[E_L(A_1),E_U(A_1)]$, $EI(A_2)=[E_L(A_2),E_U(A_2)]$ respectively. Hence, we define the Jaccard similarity in vector space as:

$$J(A_1,A_2)=\frac{[E_L(A_1)E_L(A_2)+E_U(A_1)E_U(A_2)]}{\left\{\begin{array}{c}(E_L(A_1))^2+(E_U(A_1))^2+(E_L(A_2))^2+(E_U(A_2))^2\\-[E_L(A_1)E_L(A_2)+E_U(A_1)E_U(A_2)]\end{array}\right\}}$$

$$=\frac{\begin{array}{c}w_1w_2[(a_{11}+a_{12})(a_{21}+a_{22})\\+(a_{13}+a_{14})(a_{23}+a_{24})]\end{array}}{\left\{\begin{array}{c}w_1^2[(a_{11}+a_{12})^2+(a_{13}+a_{14})^2]+w_2^2[(a_{21}+a_{22})^2\\+(a_{23}+a_{24})^2]-w_1w_2[(a_{11}+a_{12})(a_{21}+a_{22})\\+(a_{13}+a_{14})(a_{23}+a_{24})]\end{array}\right\}} \quad (1)$$

DECISION MAKING PROCEDURE BASED ON WEIGHTED EXPECTED JACCARD SIMILARITY MEASURE USING Z-NUMBER

Let $O=\{O_1,O_2,\ldots,O_m\}$ be a set of alternatives and let $C=\{C_1,C_2,\ldots,C_n\}$ be a set of criteria. Decision makers (DMs) express their rating evaluation on each alternative O_i (*i* =1, 2,..., *m*) based on the criteria set C_j (*j*=1,2,...,*n*) using the Z-number evaluation by employing the linguistic variables in the linguistic terms set (Ye, 2012c) as in Table 1 for *A* (evaluation restriction value) and Table 2 is the linguistic terms set (Kang et al., 2012) for *R* (reliability measure).

The linguistic value for linguistic terms of restriction value is represented by a trapezoidal fuzzy number (TpFN) and linguistic value for linguistic terms of reliability is represented by a triangular fuzzy number (TFN).

Table 1
Linguistic terms for evaluation (restriction values)

Linguistic terms	Linguistic values
Absolutely Low (AL)	(0.00, 0.00, 0.00, 0.00)
Very Low (VL)	(0.00, 0.00, 0.02, 0.07)
Low (L)	(0.04, 0.10, 0.18, 0.23)
Medium Low (ML)	(0.17, 0.22, 0.36, 0.42)
Medium (M)	(0.32, 0.41, 0.58, 0.65)
Medium High (MH)	(0.58, 0.63, 0.80, 0.86)
High (H)	(0.72, 0.78, 0.92, 0.97)
Very High (VH)	(0.93, 0.98, 1.00, 1.00)
Absolutely High (AH)	(1.00, 1.00, 1.00, 1.00)

Table 2
Linguistic terms for reliability measure

Linguistic terms	Linguistic values
Very Low (VL)	(0.00, 0.00, 0.25)
Low (L)	(0.00, 0.25, 0.50)
Medium (M)	(0.25, 0.50, 0.75)
High (H)	(0.50, 0.75, 1.00)
Very High (VH)	(0.75, 1.00, 1.00)

The procedure for decision making is as follows:

Step 1. Evaluation Process

Fuzzy information (fuzzy rating) as expressed by a DM which denoted by the Z-number (Z_{ij}) can be shown as in the decision matrix D below:

$$D = \begin{matrix} & \begin{matrix} C_1 & & C_n \end{matrix} \\ \begin{matrix} O_1 \\ \vdots \\ O_m \end{matrix} & \begin{bmatrix} Z_{11} & \cdots & Z_{1n} \\ \vdots & \ddots & \vdots \\ Z_{m1} & \cdots & Z_{mn} \end{bmatrix} \end{matrix}$$

where $Z_{ij} = (A_{ij}, R_{ij})$ involves two parts of the evaluation: the restriction value and the reliability of the evaluation for each alternative O_i ($i = 1,2,...,m$) with respect to each criterion C_j ($j = 1,2,...n$).

Step 2. Z-number conversion to fuzzy number (Kang et al., 2012)

Using the fuzzy expectation, all Z-numbers obtained in Step 1 are converted to fuzzy numbers as in the following procedure:

a) Convert the reliability of a Z-number, $R_{ij,}$ to a crisp value using the following equation:

$$\alpha = \frac{\int x\mu_R(x)dx}{\int \mu_R(x)dx}$$

b) Embed the converted value of R_{ij} (crisp value, α) to the first part (restriction). The evaluation is now in the form of weighted Z-number and denoted as:

$$Z^{\alpha} = \left\{ (x, \mu_{A^{\alpha}}(x) \middle| \mu_{A^{\alpha}}(x) = \alpha\mu_A(x), x \in [0,1] \right\}$$

where

$$Z_{ij}^{\alpha} = (a_{ij}{}^1, a_{ij}{}^2, a_{ij}{}^3, a_{ij}{}^4; \alpha) \quad .$$

c) Convert the weighted Z-number to a regular fuzzy number using Theorem 3 in Kang et al. (2012) as:

$$Z'_{ij} = (\sqrt{\alpha} \times a_{ij}^1, \sqrt{\alpha} \times a_{ij}^2, \sqrt{\alpha} \times a_{ij}^3, \sqrt{\alpha} \times a_{ij}^4)$$

$$= (a'^1_{ij}, a'^2_{ij}, a'^3_{ij}, a'^4_{ij})$$

Step 3. Aggregate individual evaluation to group decision matrix (Ye, 2012c)

The individual preference rating of the weighted Z-number is aggregated to the group consensus decision matrix by employing the following aggregation operator:

$$a'_{ij} = \left(\sum_{q=1}^{d} \lambda_q a'_{ijq}{}^1, \sum_{q=1}^{d} \lambda_q a'_{ijq}{}^2, \sum_{q=1}^{d} \lambda_q a'_{ijq}{}^3, \sum_{q=1}^{d} \lambda_q a'_{ijq}{}^4\right)$$

where q is a DM, d is the total number of DMs $\lambda = (\lambda_1, \lambda_2, ..., \lambda_d)$ is DMs' weight vector, $\lambda_d \in [0,1]$ and $\sum_{q=1}^{d} \lambda_q = 1$.

Step 4. Obtain the normalised decision matrix B = (b$_{ij}$)$_{mxn}$

Two types of criteria will be considered: benefit and cost . In order to transform the various criteria dimension into non-dimensional criteria, each criterion value a'_{ij} (i =1,2,..,m ; j = 1,2,..n) needs to be normalised into the corresponding comparable element in the normalised decision matrix $B = (b_{ij})_{mxn}$.

For benefit criterion: $$b_{ij} = \frac{p_j - a'_{ij}}{\sqrt{\sum_{i=1}^{m}(E(p_j - a'_{ij}))^2}}$$

And for cost criterion: $$b_{ij} = \frac{a'_{ij}}{\sqrt{\sum_{i=1}^{m}(E(a'_{ij}))^2}}$$

where $p_j = \max_{1 \le i \le m} \sup \{x_{ij} \mid \mu_{ij}(x_{ij}) > 0\}$. Thus, $B = (b_{ij})_{mxn}$.

Step 5. Determine the criteria weight, w$_j$ (Ye, 2012b)

Since the criteria weight in the MCDM problem is completely unknown, then a weight model is established based on the standard deviation of the expected values of trapezoidal fuzzy numbers in order to determine a criteria weight vector from the normalised decision matrix. The procedure is as follows:

a) Obtain the matrix of expected values from the normalised matrix B as $E = (E_{ij})_{mxn}$ where:

$$E_{ij} = \frac{b_{ij1} + b_{ij2} + b_{ij3} + b_{ij4}}{4}$$

b) Calculate the standard deviation of expected values for different alternatives with respect to a criterion as follows:

$$f_j(C_j) = \sqrt{\frac{1}{m}\sum_{i=1}^{m}\left(E_{ij} - \frac{1}{m}\sum_{i=1}^{m}E_{ij}\right)^2}$$

c) Calculate the criteria weight, w_j using weight model:

$$w_j = \frac{f_j(C_j)}{\sum_{j=1}^{n} f_j(C_j)}$$

where $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$. The weight vector is in the form $w = (w_1, w_2,...,w_n)$.

Step 6. Calculate the weighted expected Jaccard similarity measure for an alternative O_i.

A weighted expected Jaccard similarity measure between an alternative O_i and the ideal alternative O^*= (1,1,1,1), is defined as absolutely high and is calculated as:

$$W(O^*, O_i) = \sum_{j=1}^{n} w_j \frac{1.6(b_{ij}^1 + b_{ij}^2 + b_{ij}^3 + b_{ij}^4)}{5.12 + (b_{ij}^1 + b_{ij}^2)^2 + (b_{ij}^3 + b_{ij}^4)^2 - 1.6(b_{ij}^1 + b_{ij}^2 + b_{ij}^3 + b_{ij}^4)}$$

which is obtained using equation (1).

Step 7. Ranking order preferences

Rank the alternatives and select the best one(s) in accordance to the weighted expected similarity measure value. The larger the value of a weighted expected Dice similarity measure $W(O^*,O_i)$, the higher the ordering of the alternative O_i (i=1,2,...,m) in which the rank is based on the descending order.

ILLUSTRATIVE EXAMPLE

The proposed decision making procedure has been employed in investigating factors that influence customers' purchasing decision of four types of cars O_1, O_2, O_3 and O_4 which are from the same manufacturer of the family car categories with the engine cubic capacity range of 1600-2000 cc and prices in the range of RM75,000 to RM100,000. Seven decision makers (DMs) with different backgrounds in automotive industries evaluated each alternative with respect to 15 criteria which are value for money (C_1), fuel consumption (C_2), power (C_3), riding comfort (C_4), performance (C_5), safety (C_6), equipment and interior (C_7), exterior design/size (C_8), after sales maintenance (C_9), environmental friendly (C_{10}), brand image (C_{11}), resale value (C_{12}), advancing technology (C_{13}), delivery time (C_{14}) and promotion (C_{15}).

Evaluations are given by the seven DMs using linguistic terms (for restriction value) and confidence level (reliability) as shown in Table 1 and Table 2 respectively. Table 3, 4, 5 and 6 show the full evaluations given by the decision makers in the form of Z-numbers.

Table 3
Evaluation by DMs on O_1

	DM1	DM2	DM3	DM4	DM5	DM6	DM7
C1	(MH,M)	(VH,VH)	(M,M)	(MH,H)	(MH,H)	(L,H)	(MH,H)
C2	(MH,H)	(L,VH)	(H,H)	(MH,H)	(MH,VH)	(L,H)	(H,H)
C3	(MH,H)	(L,VH)	(ML,M)	(H,H)	(ML,VH)	(ML,H)	(VH,H)
C4	(H,VH)	(ML,VH)	(H,H)	(M,H)	(ML,VH)	(M,L)	(VH,H)
C5	(M,VH)	(ML.VH)	(M,H)	(MH,H)	(M,VH)	(ML,L)	(VH,H)
C6	(H,H)	(H,VH)	(MH,H)	(MH,H)	(M,VH)	(M,L)	(AH,H)
C7	(MH,VH)	(L,VH)	(M,,H)	(M.H)	(M,H)	(ML,H)	(VH,H)
C8	(AH,VH)	(H,VH)	(M.H)	(MH,H)	(M,VH)	(ML,L)	(VH,H)
C9	(VH,VH)	(M,VH)	(H,H)	(MH,H)	(H,H)	(M,L)	(VH,H)
C10	(VH,H)	(M,VH)	(ML,H)	(H,H)	(ML,VH)	(M,L)	(AH,H)
C11	(MH,VH)	(H,VH)	(H,H)	(M,H)	(H,VH)	(ML,M)	(AH,H)
C12	(HH,M)	(VH,VH)	(H,H)	(MH,H)	(H,VH)	(ML,L)	(VH,H)
C13	(MH,VH)	(M,VH)	(M,H)	(M,H)	(MH,VH)	(L,M)	(H,H)
C14	(M,H)	(M.VH)	(H,H)	(M,H)	(MH,VH)	(H,H)	(AH,H)
C15	(MH,M)	(M,VH)	(H,H)	(MH,H)	(H,VH)	(L,H)	(H,H)

Table 4
Evaluation by DMs on O_2

	DM1	DM2	DM3	DM4	DM5	DM6	DM7
C1	(H,H)	(AH,VH)	(M,M)	(VH,H)	(H,VH)	(H,H)	(H,H)
C2	(H,H)	(L,VH)	(H,H)	(H,H)	(H,VH)	(ML,H)	(VH,H)
C3	(VH,H)	(MH,VH)	(H,H)	(H,H)	(H,VH)	(MH,H)	(VH,H)
C4	(H,H)	(VH,VH)	(M,H)	(H,H)	(VH,VH)	(H,M)	(AH,H)
C5	(H,VH)	(M.VH)	(H,H)	(H,H)	(H,VH)	(MH,H)	(AH,H)
C6	(H,H)	(H,VH)	(MH,H)	(H,H)	(VH,VH)	(VH,H)	(AH,H)
C7	(H,H)	(MH,VH)	(M,H)	(H.H)	(AH,VH)	(M,H)	(VH,H)
C8	(H,H)	(H,VH)	(M.H)	(H,H)	(AH,VH)	(H,H)	(VH,H)
C9	(VH,H)	(M,VH)	(H,H)	(H,H)	(H,H)	(MH,H)	(VH,H)
C10	(VH,H)	(M,VH)	(ML,H)	(H,H)	(H,H)	(M,H)	(AH,H)
C11	(H,VH)	(H,VH)	(H,H)	(H,H)	(H,VH)	(H,H)	(AH,H)
C12	(MH,VH)	(VH,VH)	(H,H)	(H,H)	(H,VH)	(H,H)	(VH,H)
C13	(H,H)	(H,VH)	(M,H)	(MH,H)	(MH,VH)	(M,H)	(H,H)
C14	(H,M)	(M.VH)	(H,H)	(M,H)	(H,H)	(M,H)	(H,H)
C15	(VH,H)	(M,VH)	(H,H)	(MH,H)	(MH,H)	(M,H)	(H,H)

Table 5
Evaluation by DMs on O_3

	DM1	**DM2**	**DM3**	**DM4**	**DM5**	**DM6**	**DM7**
C1	(MH,M)	(H,VH)	(M,M)	(MH,H)	(MH,H)	(ML,H)	(MH,H)
C2	(MH,H)	(L,VH)	(H,H)	(M,H)	(ML,H)	(L,H)	(H,H)
C3	(H,H)	(L,VH)	(ML,M)	(H,H)	(ML,H)	(ML,H)	(VH,H)
C4	(H,H)	(M,VH)	(H,H)	(MH,H)	(ML,H)	(M,L)	(VH,H)
C5	(H,M)	(L.VH)	(M,M)	(H,H)	(M,VH)	(M,L)	(VH,H)
C6	(H,H)	(H,VH)	(MH,H)	(H,H)	(M,VH)	(M,M)	(AH,H)
C7	(MH,H)	(ML,VH)	(M,H)	(MH.H)	(M,VH)	(ML,L)	(VH,H)
C8	(M,H)	(L,VH)	(M.H)	(H,H)	(M,VH)	(L,H)	(VH,H)
C9	(VH,H)	(M,VH)	(H,H)	(H,H)	(H,H)	(M,L)	(VH,H)
C10	(H,H)	(H,VH)	(H,H)	(MH,H)	(MH,VH)	(M,M)	(AH,H)
C11	(H,VH)	(H,VH)	(H,H)	(H,H)	(H,VH)	(H,H)	(AH,H)
C12	(MH,M)	(VH,VH)	(H,H)	(MH,H)	(H,H)	(M,L)	(VH,H)
C13	(H,M)	(L,VH)	(M,H)	(MH,H)	(MH,VH)	(M,H)	(H,H)
C14	(M,M)	(M.VH)	(H,H)	(M,H)	(H,H)	(M,H)	(AH,H)
C15	(H,VH)	(M,VH)	(H,H)	(MH,H)	(MH,H)	(L,H)	(H,H)

Table 6
Evaluation by DMs on O_4

	DM1	**DM2**	**DM3**	**DM4**	**DM5**	**DM6**	**DM7**
C1	(AH,VH)	(AH,VH)	(H,H)	(VH,H)	(H,VH)	(M,H)	(VH,H)
C2	(VH,VH)	(VL,VH)	(H,H)	(H,H)	(H,VH)	(H,H)	(VH,H)
C3	(AH,VH)	(M,VH)	(M,M)	(H,H)	(H,VH)	(M,H)	(VH,H)
C4	(AH,VH)	(H,VH)	(H,H)	(MH,H)	(VH,VH)	(H,H)	(VH,H)
C5	(AH,VH)	(M.VH)	(H,H)	(H,H)	(H,VH)	(MH,H)	(VH,H)
C6	(H,VH)	(H,VH)	(MH,H)	(MH,H)	(MH,VH)	(VH,H)	(AH,H)
C7	(AH,VH)	(M,VH)	(M,,H)	(MH.H)	(M,H)	(ML,H)	(VH,H)
C8	(AH,VH)	(H,VH)	(M.H)	(MH,H)	(VH,VH)	(H,H)	(VH,H)
C9	(AH,VH)	(M,VH)	(H,H)	(H,H)	(H,H)	(H,H)	(VH,H)
C10	(AH,VH)	(M,VH)	(ML,H)	(H,H)	(H,H)	(M,H)	(AH,H)
C11	(AH,VH)	(H,VH)	(H,H)	(H,H)	(VH,H)	(H,H)	(AH,H)
C12	(VH,VH)	(VH,VH)	(H,H)	(H,H)	(VH,H)	(H,H)	(VH,H)
C13	(AH,VH)	(M,VH)	(M,H)	(M,H)	(M,H)	(AH,H)	(VH,H)
C14	(VH,VH)	(M.VH)	(H,H)	(M,H)	(MH,H)	(M,H)	(AH,H)
C15	(AH,VH)	(M,VH)	(H,H)	(MH,H)	(MH,H)	(M,H)	(VH,H)

Using equation in Step 5 of the procedure, the criteria weights C_j (j =1,2,..., 15) are obtained as the following weight vector $w_j = (w_1, w_2,..., w_{15})$.

$w_1 = 0.1003$, $w_2 = 0.0651$, $w_3 =0.0969$, $w_4 =0.0960$,
$w_5 = 0.0850$, $w_6 = 0.0426$, $w_7 =0.0892$, $w_8 = 0.0939$,
$w_9 = 0.0453$, $w_{10} = 0.0474$, $w_{11} = 0.0431$, $w_{12} = 0.0500$,
$w_{13} = 0.0639$, $w_{14} = 0.0421$, $w_{15} = 0.0391$.

A weighted expected Jaccard similarity measure between an alternative O_i and the ideal alternative O^* is calculated as

$W (O^*, O_1) = 0.5906$ W, $W (O^*, O_2) = 0.7948$,
$W (O^*, O_3) = 0.5963$, $(O^*, O_4) = 0.7967$.

Hence, the ranking of the alternatives of cars with respect to the 15 criteria under consideration is as follows:

$O_4 > O_2 > O_3 > O_1$.

From the ranking order, the most favoured car is O_4. Moreover, from the criteria weight vector, the most influential factor in influencing decision to purchase a car is its value for money while the least important is the promotion factor.

CONCLUSION

Decision making in a fuzzy environment leads to vague expressions of evaluation. The fuzzy set theory has the ability via fuzzy numbers to represent vague phrases or languages which involve subjectivity of values. Utilising the Z-number concept, decision making process with the expected Jaccard similarity measure technique provides a better decision, owing to the inclusion of the reliability factor, particularly in fuzzy evaluation of multi-criteria decision making problem. The reliability factor in the Z-number representation is a useful tool in the evaluation process as the background of the decision makers usually defers to one another in terms of experience, knowledge, authority level and so on. As an extension of this effort, different similarity measures may be used in the proposed procedure and comparison of results may be made. Furthermore, the proposed procedure can be applied in other decision making problems in various areas with fuzzy environment that also considers the level of confidence of evaluators.

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